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RESEARCH ARTICLE

FURTHER EXTENDED GAMMA AND BETA FUNCTIONS IN TERMS OF GENERALIZED WRIGHT FUNCTION

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Abstract

The main objective of this paper is to introduce a new extension of extended Gamma and Beta functions in terms of generalized Wright function. Various properties of these extended functions are investigated such as integral representations, summation formulas and Mellin transform.

Keywords: Extended Beta function, Wright function, integral representations, summation formula.

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1. Introduction

Many extensions of the different special functions (the Gamma and Beta functions, the Gauss hypergeometric function, and so on) have been introduced by different authors (see **[1]**, **[3, 4]**, **[8-10]**).

$$^{\psi}\Gamma_p^{(\alpha,\beta)}(x) = \int_0^\infty t^{x-1}\ {}_1\psi_1\left(\alpha,\beta;-t-\frac{p}{t}\right)dt, \tag{1.1}$$

$$(Re(x)>0, Re(\alpha)>0, Re(\beta)>1, Re(p)>0),$$

$$^{\psi}B_p^{(\alpha,\beta)}(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\ {}_1\psi_1\left(\alpha,\beta;-\frac{p}{t(1-t)}\right)dt, \tag{1.2}$$

$$\left(\begin{matrix}Re(x)>0, Re(y)>0\ , Re(\alpha)>0, Re(\beta)>1,\\ Re(p)>0\end{matrix}\right),$$

where ${}_1\psi_1(.)$ denoted the Wright function defined by **[11]**

$${}_1\psi_1(\alpha,\beta;z) = \sum_{n=0}^\infty \frac{1}{\Gamma(\alpha n+\beta)}\frac{z^n}{n!}, \tag{1.3}$$

For $\alpha = 0$ and $\beta = 2$, equations (1.1) and (1.2) reduce to the extended Gamma and Beta type functions due to Chaudhry et al. [4] defined by

$$\Gamma_p(x) = \int_0^\infty t^{x-1}exp\left(-t-\frac{p}{t}\right)dt, \tag{1.4}$$

$$(Re(p)>0).$$

$$B(x,y;p) = \int_0^1 t^{x-1}(1-t)^{y-1}e^{\frac{-p}{t(1-t)}}dt, \tag{1.5}$$

$$(where\ Re(p)>0, Re(x)>0, Re(y)>0).$$

Which for $p = 0$ give the classical Gamma and Beta functions defined by **[4]**

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt, \qquad (Re(x)>0), \tag{1.6}$$

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt, \tag{1.7}$$

$$(Re(x)>0, Re(y)>0\ ),$$

In this paper, we introduce the following extension of Gamma and Euler's Beta functions:

$$^{W}\Gamma_p^{(\alpha,\beta;\gamma,\delta)}(x) = \int_0^\infty t^{x-1}W_{\alpha,\beta}^{\gamma,\delta}\left(-t-\frac{p}{t}\right)dt, \tag{1.8}$$

$$\left(\begin{matrix}Re(x)>0, \alpha,\beta,\gamma,\delta\in C; \alpha>-1, \delta\neq 0,-1,-2,\dots\\ p\geq 0\end{matrix}\right)$$

$$^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\, W_{\alpha,\beta}^{\gamma,\delta}\left(-\frac{p}{t(1-t)}\right) dt, \quad (1.9)$$

where $Re(x) > 0, Re(y) > 0, \alpha,\beta,\gamma,\delta \in C; \alpha > -1, \delta \neq 0,-1,-2,\ldots p \geq 0,$

and $W_{\alpha,\beta}^{\gamma,\delta}(.)$ denotes the generalized Wright function defined by [5]

$$W_{\alpha,\beta}^{\gamma,\delta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{(\delta)_n \Gamma(\alpha n+\beta)} \frac{z^n}{n!}. \quad (1.10)$$

$$\left(\begin{matrix}\alpha,\beta,\gamma,\delta \in C; \alpha > -1, \delta \neq 0,-1,-2,\ldots, z \in C \text{ and} \\ |z| < 1, with\ \alpha = -1\end{matrix}\right),$$

It is obvious that,

$$^{W}\Gamma_p^{(\alpha,\beta;\delta,\delta)}(x) = {}^{\Psi}\Gamma_p^{(\alpha,\beta)}(x),$$

$$^{\Psi}\Gamma_p^{(0,2)}(x) = \Gamma_p(x),$$

$$^{\Psi}\Gamma_0^{(0,2)}(x) = \Gamma(x),$$

$$^{W}B_p^{(\alpha,\beta;\delta,\delta)}(x,y) = {}^{\Psi}B_p^{(\alpha,\beta)}(x,y),$$

$$^{\Psi}B_p^{(0,2)}(x,y) = B_p(x,y),$$

$$^{\Psi}B_0^{(0,2)}(x,y) = B(x,y).$$

2. Properties of Extended Gamma and Beta functions

In this section, we introduce some properties of the new extended Gamma and Beta functions in the form of the following theorems:

Theorem 2.1. For the product of two Gamma function $^{W}\Gamma_p^{(\alpha,\beta;\gamma,\delta)}(.)$, we have the following integral representation:

$$^{W}\Gamma_p^{(\alpha,\beta;\gamma,\delta)}(x)\,{}^{W}\Gamma_p^{(\alpha,\beta;\gamma,\delta)}(y)$$

$$= 4\int_0^{\frac{\pi}{2}}\int_0^{\infty} r^{2(x+y)-1} cos^{2x-1}\theta\, sin^{2y-1}\theta$$

$$\times W_{\alpha,\beta}^{\gamma,\delta}\left(-r^2 cos^2\theta - \frac{p}{r^2 cos^2\theta}\right)$$

$$\times W_{\alpha,\beta}^{\gamma,\delta}\left(-r^2 sin^2\theta - \frac{p}{r^2 sin^2\theta}\right) dr d\theta. \quad (2.1)$$

Proof. Substituting $t = \eta^2$ in definition (1.8), we get

$$^{W}\Gamma_p^{(\alpha,\beta;\gamma,\delta)}(x) = 2\int_0^{\infty} \eta^{2x-1}\, W_{\alpha,\beta}^{\gamma,\delta}\left(-\eta^2 - \frac{p}{\eta^2}\right) d\eta,$$

Now, we have

$$^{W}\Gamma_p^{(\alpha,\beta;\gamma,\delta)}(x)\,{}^{W}\Gamma_p^{(\alpha,\beta;\gamma,\delta)}(y)$$

$$= 4\int_0^{\infty}\int_0^{\infty} \eta^{2x-1}\xi^{2y-1}\, W_{\alpha,\beta}^{\gamma,\delta}\left(-\eta^2 - \frac{p}{\eta^2}\right)$$

$$\times W_{\alpha,\beta}^{\gamma,\delta}\left(-\xi^2 - \frac{p}{\xi^2}\right) d\eta d\xi,$$

Which on putting $\eta = rcos\,\theta$ and $\xi = rsin\theta$, we get the desired result.

Theorem 2.2. The new extended of Beta function has the following relation:

$$^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x+1,y) + {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y+1)$$

$$= {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y) \quad (2.2)$$

Proof. Consider the left hand side of (2.2), we have

$$^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x+1,y) + {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y+1)$$

$$= \int_0^1 \{t^x(1-t)^{y-1} + t^{x-1}(1-t)^y\} \times W_{\alpha,\beta}^{\gamma,\delta}\left(-\frac{p}{t(1-t)}\right) dt,$$

$$= \int_0^1 t^{x-1}(1-t)^{y-1}\{t+(1-t)\}\, W_{\alpha,\beta}^{\gamma,\delta}\left(-\frac{p}{t(1-t)}\right) dt,$$

$$= \int_0^1 t^{x-1}(1-t)^{y-1}\, W_{\alpha,\beta}^{\gamma,\delta}\left(-\frac{p}{t(1-t)}\right) dt,$$

Which proves the desired result.

Theorem 2.3. The new extended of Beta function has the following summation formula:

$$^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,1-y)$$

$$= \sum_{n=0}^{\infty} \frac{(y)_n}{n!}\, {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x+n,1). \quad (2.3)$$

Proof. Consider the generalized binomial theorem

$$(1-t)^{-y} = \sum_{n=0}^{\infty} \frac{(y)_n}{n!} t^n, \qquad (|t| < 1). \quad (2.4)$$

Applying (2.4) to the definition (1.9) of extended Beta function, we get

$$^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,1-y)$$

$$= \int_0^1 \sum_{n=0}^{\infty} \frac{(y)_n}{n!} t^{x+n-1} W_{\alpha,\beta}^{\gamma,\delta}\left(-\frac{p}{t(1-t)}\right) dt,$$

Now, interchanging the order of summation and integration in above equation and using (1.9), we get desired result.

Theorem 2.4. The new extended of Beta function has the following summation
Formula:

$$ {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y) = \sum_{n=0}^{\infty} {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x+n,y+1). \quad (2.5)$$

Proof. Replacing the following series representation

$$(1-t)^{y-1} = (1-t)^{y}\sum_{n=0}^{\infty} t^{n} \ ,$$

In definition(2.1), we obtain

$$ {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y)$$

$$= \int_0^1 (1-t)^{y} \sum_{n=0}^{\infty} t^{x+n-1} W_{\alpha,\beta}^{\gamma,\delta}\left(-\frac{p}{t(1-t)}\right) dt,$$

Interchanging the order of integration and summation in above equation and using (1.9), we get the desired result.

Theorem 2.5. The following relation holds true

$$ {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y)$$

$$= \sum_{k=0}^{n} \binom{n}{k} {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x+k,y+n-k), \qquad (2.6)$$

$$(n \in \mathbb{N}_0).$$

Proof. Starting with $n = 1, 2, 3, \ldots$, in (2.6) we have:
For $n = 1$, we get

$$ {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y)$$

$$= {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y+1) + {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x+1,y),$$

For $n = 2$, we get

$$ {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y)$$

$$= {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x+2,y) + 2\,{}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x+1,y+1)$$

$$+ {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y+2),$$

For $n = 3$, we get

$$ {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y)$$

$$= {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y+3) + 3\,{}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x+1,y+2)$$

$$+ 3\,{}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x+2,y+1) + {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x+3,y),$$

and so on. The above series behaves like as finite binomials series does. Thus, we can finally obtain the desired relation (2.6).

Theorem 2.6. The new extended Beta function has the following summation formula:

$$ {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y)$$

$$= \sum_{m=0}^{\infty} \frac{(\gamma)_m(-p)^m}{(\delta)_m \Gamma(\alpha m+\beta) m!} B(x-m,y-m). \quad (2.7)$$

Proof. Expanding the Wright functions in the right hand said of equation (1.9) and then using relation (1.7) in the resultant equation, we get the desired result.

Theorem 2.7. The new extended Beta function has the following summation formula:

$$ {}^{W}B_p^{(\alpha,\beta,\gamma,\gamma+n+1)}(x,y)$$

$$= (\gamma)_{n+1} \sum_{k=0}^{n} \frac{(-1)^k}{k!\,(n-k)!\,(\gamma+k)}\ {}^{W}B_p^{(\alpha,\beta,\gamma+k,\gamma+k+1)}(x,y). \quad (2.8)$$

Proof. Consider the following relation [5. P. 8(71)]:

$$W_{\alpha,\beta}^{\gamma,\gamma+n+1}(z)$$

$$= (\gamma)_{n+1} \sum_{k=0}^{n} \frac{(-1)^k}{k!\,(n-k)!\,(\gamma+k)}\ W_{\alpha,\beta}^{\gamma+k,\gamma+k+1}(z),$$

Replacing z by $\frac{-p}{t(1-t)}$ in the above equation and multiplying both sides of the resultant equation by $t^{x-1}(1-t)^{y-1}$ and then integrating with respect to t from 0 to 1 we obtain

$$\int_0^1 t^{x-1}(1-t)^{y-1} W_{\alpha,\beta}^{\gamma,\gamma+n+1}\left(\frac{-p}{t(1-t)}\right) dt$$

$$= (\gamma)_{n+1} \sum_{k=0}^{n} \frac{(-1)^k}{k!\,(n-k)!\,(\gamma+k)}$$

$$\times \int_0^1 t^{x-1}(1-t)^{y-1} W_{\alpha,\beta}^{\gamma+k,\gamma+k+1}\left(\frac{-p}{t(1-t)}\right) dt,$$

which on using definition (1.9), we get the desired result.

Remark 2.1. For *n=1* in result (2.8), we get the following recurrence relation:

$$ {}^{W}B_p^{(\alpha,\beta;\gamma,\gamma+2)}(x,y)$$

$$= (\gamma+1)\,{}^{W}B_p^{(\alpha,\beta;\gamma,\gamma+1)}(x,y) + \gamma\,{}^{W}B_p^{(\alpha,\beta;\gamma+1,\gamma+2)}(x,y). \quad (2.9)$$

Theorem 2.8. The new extended Beta function has the following summation formula:

$$ {}^{W}B_p^{(\alpha,\beta-1,\gamma,\delta)}(x,y) + (1-\beta)\,{}^{W}B_p^{(\alpha,\beta,\gamma,\delta)}(x,y) $$

$$ = \frac{-\alpha\gamma p}{\delta}\,{}^{W}B_p^{(\alpha,\alpha+\beta,\gamma+1,\delta+1)}(x-1,y-1). \qquad (2.10) $$

Proof. Consider the following relation [5. P. 9(74)]:

$$ W_{\alpha,\beta-1}^{\gamma,\delta}(z) + (1-\beta)W_{\alpha,\beta}^{\gamma,\delta}(z) = \frac{\alpha\gamma z}{\delta} W_{\alpha,\alpha+\beta}^{\gamma+1,\delta+1}(z)\,, $$

Replacing z by $\frac{-p}{t(1-t)}$ in the above equation and multiplying both sides of the resultant equation by $t^{x-1}(1-t)^{y-1}$ and then integrating with respect to t from 0 to 1 we obtain

$$ \int_0^1 t^{x-1}(1-t)^{y-1} W_{\alpha,\beta-1}^{\gamma,\delta}\left(\frac{-p}{t(1-t)}\right)dt + (1-\beta) $$

$$ \times \int_0^1 t^{x-1}(1-t)^{y-1} W_{\alpha,\beta}^{\gamma,\delta}\left(\frac{-p}{t(1-t)}\right)dt $$

$$ = \frac{-\alpha\gamma p}{\delta}\int_0^1 t^{x-2}(1-t)^{y-2} W_{\alpha,\alpha+\beta}^{\gamma+1,\delta+1}\left(\frac{-p}{t(1-t)}\right)dt $$

which on using definition (1.9), we get the desired result.

3. Integral Formulas

In this section, we get some integral formulas for the new extended Beta function in form of the following theorems:

Theorem 3.1. The new extended of extended Beta function has the following Mellin transform relation:

$$ \mathcal{M}\left\{{}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y); p \longrightarrow s\right\} $$

$$ = B(x+s,y+s)\,{}^{W}\Gamma_0^{(\alpha,\beta;\gamma,\delta)}(s), \qquad (3.1) $$

$$ Re(x+s) > 0, Re(y+s) > 0\,, Re(\alpha) > 0, $$

$$ Re(\beta) > 1, Re(p) > 0, Re(s) > 0. $$

Proof. By applying the Mellin transform to (1.9)

$$ \mathcal{M}\left\{{}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y); p \longrightarrow s\right\} $$

$$ = \int_0^\infty p^{s-1}\int_0^1 t^{x-1}(1-t)^{y-1}\, W_{\alpha,\beta}^{\gamma,\delta}\left(-\frac{p}{t(1-t)}\right)dtdp. \; (3.2) $$

Interchanging the order of integrations, we have

$$ \mathcal{M}\left\{{}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y); p \longrightarrow s\right\} $$

$$ = \int_0^1 t^{x-1}(1-t)^{y-1} $$

$$ \times\left\{\int_0^\infty p^{s-1} W_{\alpha,\beta}^{\gamma,\delta}\left(-\frac{p}{t(1-t)}\right)dp\right\}dt, \qquad 3.3) $$

Substituting $v = \frac{p}{t(1-t)}$ in (3.3), we get

$$ \mathcal{M}\left\{{}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y); p \longrightarrow s\right\} $$

$$ = \int_0^1 t^{x+s-1}(1-t)^{y+s-1}dt\left\{\int_0^\infty v^{s-1}W_{\alpha,\beta}^{\gamma,\delta}(-v)dv\right\}. \; (3.4) $$

Using definition (1.1) (for $p = 0$)in the right hand said of the above equation, we get the desired result.

Theorem 3.2. The new extended of Beta function has the following Mellin transforms formula:

$$ {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y) $$

$$ = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{\Gamma(x+s)\Gamma(y+s)\,{}^{W}\Gamma_0^{(\alpha,\beta;\gamma,\delta)}(s)}{\Gamma(x+y+2s)}\,p^{-s}ds, \; (3.5) $$

$$ (Re(x) > 0, Re(y) > 0\,, Re(\alpha) > 0, Re(\beta) > 1, p \geq 0,). $$

Proof. Applying the inverse Mellin transform on both sides of (3.1), we get the desired result.

Theorem 3.3. The following integral representations holds true:

$$ {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y) $$

$$ = 2\int_0^{\frac{\pi}{2}} cos^{2x-1}\theta sin^{2y-1}\,\theta W_{\alpha,\beta}^{\gamma,\delta}\left(-\frac{p}{cos^2\theta\, sin^2\theta}\right)d\theta, \quad (3.6) $$

$$ {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y) $$

$$ = \int_0^\infty \frac{u^{x-1}}{(1+u)^{x+y}} W_{\alpha,\beta}^{\gamma,\delta}\left(-2p - p\left(u+\frac{1}{u}\right)\right)du, \qquad (3.7) $$

$$ {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y) $$

$$ = (c-a)^{1-x-y}\int_a^c (u-a)^{x-1}(c-u)^{y-1} $$

$$ \times W_{\alpha,\beta}^{\gamma,\delta}\left(\frac{-p(c-a)^2}{(u-a)(c-u)}\right)du. \qquad (3.8) $$

Proof. Equations (3.6) - (3.8) can be easily obtained by taking the transformation $t = cos^2\,\theta,\ t = \frac{u}{1+u}$ and $t = \frac{u-a}{c-a}$ in (2.9), respectively.

Theorem 3.4. The following integral representations holds true:

$$ {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y) = \frac{\Gamma(\delta)}{\Gamma(\gamma)} \int_0^1 t^{x-1}(1-t)^{y-1} \times H_{1\ 3}^{1\ 1}\left[\frac{p}{t(1-t)}\middle|\begin{matrix}(1-\gamma,1)\\(0,1),(1-\beta,\alpha),(1-\delta,1)\end{matrix}\right], \quad (3.9)$$

$$ {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y) = \frac{\Gamma(\delta)}{\Gamma(\gamma)} \int_0^1 t^{x-1}(1-t)^{y-1} \times {}_1\psi_2\left[\begin{matrix}(\gamma,1) & ; \\ (\delta,1),(\beta,\alpha); & \end{matrix}\frac{-p}{t(1-t)}\right]dt, \quad (3.10)$$

$$ {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y) = \frac{\Gamma(\delta)}{\Gamma(\gamma)} \int_0^1 t^{x-1}(1-t)^{y-1} \times G_{1\ 3}^{1\ 1}\left[\frac{p}{t(1-t)}\middle|\begin{matrix}1-\gamma\\0,1-\beta,1-\gamma\end{matrix}\right]dt. \quad (3.11)$$

where $H_{p\ q}^{m\ n}(.)$denoted the Fox H-function [7], ${}_2\psi_1$denoted the Fox-Wright function [6] and $G_{p\ q}^{m\ n}(.)$ denoted the Meijer G-Function [2].

Proof. Applying the following relations :

$$W_{\alpha,\beta}^{\gamma,\delta}(-z) = \frac{\Gamma(\delta)}{\Gamma(\gamma)} H_{1\ 3}^{1\ 1}\left[z\middle|\begin{matrix}(1-\gamma,1)\\(0,1),(1-\beta,\alpha),(1-\delta,1)\end{matrix}\right],$$

$$W_{\alpha,\beta}^{\gamma,\delta}(z) = \frac{\Gamma(\delta)}{\Gamma(\gamma)}\ {}_1\psi_2\left[\begin{matrix}(\gamma,1) & ; \\ (\delta,1),(\beta,\alpha); & \end{matrix} z\right],$$

$$W_{\alpha,\beta}^{\gamma,\delta}(-z) = \frac{\Gamma(\delta)}{\Gamma(\gamma)} G_{1\ 3}^{1\ 1}\left[z\middle|\begin{matrix}1-\gamma\\0,1-\beta,1-\gamma\end{matrix}\right].$$

in the R.H.S. of equation (1.9) respectively, we get the desired results.

Theorem 3.5. The following integral representations holds true:

$$ {}^{W}B_p^{(\alpha,\beta;\gamma,\delta)}(x,y) = \frac{1}{B(\gamma,\delta-\gamma)} \int_0^1 \int_0^1 u^{\gamma-1}(1-u)^{\delta-\gamma-1} t^{x-1}(1-t)^{y-1} \times W_{\alpha,\beta}\left(-\frac{pu}{t(1-t)}\right) du dt. \quad (3.12)$$

Proof. Applying the following relations:

$$W_{\alpha,\beta}^{\gamma,\delta}(z) = \frac{\Gamma(\delta)}{\Gamma(\gamma)\Gamma(\delta-\gamma)} \int_0^1 u^{\gamma-1}(1-u)^{\delta-\gamma-1} W_{\alpha,\beta}(zu)du.$$

in the R.H.S. of equation (1.9) , we get the desired results.

References

[1] Al-Gonah, A. A., Mohammed, W. K.(2020)."Certain extension of the Hurwitz- Lerch zeta function and its properties". Electronic Journal of University of Aden For Basic and Applied Sciences. 1(1): 39-48.

[2] Andrews, L. C. (1985). Special Functions for Engineers Mathematicians. MacMillan, New York, NY. USA.

[3] Ata, E. (2018). "Generalized Beta Function Defined by Wright Function", preprint". arXiv:1803.03121v1 [math.CA].

[4] Chaudhry, M. A., Qadir, A., Rafique, M. and Zubair, S. M. (1997). "Extension of Eulers Beta function " .J. Comput. Appl. Math. 78:19-32.

[5] El-Shahed, M. and Salem, A. (2015)."An extension of wright function and properties". Hindawi Publishing Corporation. J.Math.2015 (Article 2015):1-11.

[6] Luchko, Y. (2009). "Maximum principle for the generalized time fractional diffusion equation". Journal of Mathematical Analysis and Applications. 351(1): 218–223.

[7] Mathai, A. M. Saxena, R. K. and Haubold, J. J. (2010). "The H-Function: Theory and Applications". Springer, Dordrecht. The Netherlands.

[8] Mohsen, F.B.F. and Kulib, M. A. H.(2016). "Some properties of Generalized Gamma and Beta functions". Aden university Journal of Natural and Applied Sciences.20(2): 365-374.

[9] Ozergin, E., Ozarslan, M. A. and Altin, A. (2011). "Extension of Gamma, Beta and hypergeometric functions". J. comput. Appl. Math. 235(16): 4601-4610.

[10] Parmar, R. K. (2013)."A new generalization of Gamma, Beta hypergeometric and confluent hypergeometric functions". Le Matematiche. LXVIII(II): 33-52.

[11] Rainville, E.D. (1960). Special functions. Chelsea Pub.Com. New York.

مقالة بحثية

تمديدات إضافية لدوال جاما وبيتا بدلالة دالة رايت المعممة

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المُلخّص

الهدف الرئيسي من هذه البحث إلى تقديم تمديد جديد لدوال جاما وبيتا الممددة بدلالة دالة رايت المعممة. كما تم إيجاد عدد من الخواص لهذه الدوال الممددة مثل التمثيلات التكاملية وصيغ الجمع وتحويل ميلن.

الكلمات الرّئيسية: دالة بيتا الممددة، دالة رايت، تمثيلات تكاملية، صيغ مجاميع.